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You Are Known by the Directors You Keep: Reputable Directors as a Signaling Mechanism for Young Firms

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In this paper, we develop an analytical model of outside directors’ signaling role—a role that is especially important for entrepreneurial firms. We formally demonstrate that in the face of a market failure in which stakeholders refuse to align themselves with new firms, high-quality new ventures may be able to credibly signal their type by appointing reputable directors to their boards. However, this option is not universally feasible. Both directors’ reputations and the quality of their information determine the effectiveness of this strategy. In contrast to earlier adverse selection models, we demonstrate that when the middlemen (directors) have incomplete information on firm quality, bad and good firms can coexist in equilibrium. In this equilibrium, the quality of the directors’ information determines the mix of good and bad firms in the population of surviving firms. Avenues for future research and normative implications for practitioners are discussed.

(Reputable Directors; Signaling Firm Quality; New Ventures)

1. Introduction
Directors’ principal functions can be classified into two broadly defined roles: one internally focused and the other externally directed (Finkelstein and Hambrick 1996). Internally, directors are responsible for policy setting and monitoring managers’ actions (Fama 1980, Fama and Jensen 1983, Mizruchi 1983). Externally, directors facilitate better access to important resources in the firm’s environment (Pfeffer 1972, Pfeffer and Salancik 1978). The latter role is particularly important to entrepreneurial firms (Daily and Dalton 1992, 1993) for which establishing ties with other entities in their environment is typically more difficult than for established firms (Pfeffer and Salancik 1978). As argued by Stinchcombe (1965), the inability of young organizations to develop relations with potential stakeholders is a major determinant of their relatively high mortality rate or the “liability of newness.”

Outside directors can facilitate access to resources in the firm’s environment not only by providing direct connections to important entities (e.g., Mizruchi 1996), but also, in some contexts, by playing a symbolic role that is independent of their tangible activities (Certo 2003). Specifically, reputable outside directors can help establish the legitimacy the young firm often lacks by acting as signaling mechanism (Selznick 1949, Pfeffer and Salancik 1978). These prestigious individuals can provide cues to the firm’s environment regarding different aspects of the firm, such as its social responsibility (Schoorman et al. 1981), wealth, and value (Pfeffer and Salancik 1978).

While the literature has recognized the benefits young firms can gain by appointing reputable outside directors to signal their quality (e.g., Certo 2003,
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Finkel 1998, Johnson et al. 1996, Provan 1980), to date no previous research has formally modeled this signaling role. This paper aims to advance the literature on directors’ signaling role by developing an analytical model, in which superior entrepreneurial firms “rent” the reputation of prestigious outside directors to separate themselves from lesser types in the attempt to attract stakeholders crucial for their survival. This model extends previous research on directors’ signaling role in several ways. First, we formally show that young firms can credibly signal their attributes through the appointment of reputable outside directors. Second, we derive conditions under which signaling is feasible. Third, we analyze how the quality of directors’ information influences the effectiveness of this strategy. Finally, we examine determinants of the optimal level of directors’ reputational capital rented by the firm.

2. Renting a Reputation

Economic agents frequently must contract with each other without full information about important characteristics of the other party. For example, buyers may not know the quality of the product a seller offers, employees may be uncertain about the quality of a prospective employer’s work environment, and investors may have little knowledge of the likely profitability of a new company. These informational asymmetries may lead to market failure (Akerlof 1970).

When markets are vulnerable to this kind of failure, it is in many parties’ interests to find ways to communicate the missing information. In particular, firms that are “better” than average would benefit if their potential stakeholders had this information. The literature on asymmetric information has suggested a number of mechanisms to correct this problem, including the use of reputational mechanisms (Spence 1974). Stakeholders, for example, often rely on a firm’s reputation when making investment, product, and career decisions (Dowling 1986).

Reputation can be thought of as a probability that the firm is of a certain type or will act in a certain way (e.g., Shapiro 1983, Tadelis 1999). Seen this way, firms can develop reputations for many aspects others may care about. For example, a firm may be seen as having a reputation for high-quality products, poor labor relations, or questionable environmental practices. Because a firm’s reputation determines how willing others are to deal with the firm (and on what terms), the firm cares a great deal about cultivating the right reputation (Weigelt and Camerer 1988).

Most often, today’s reputation is built on yesterday’s actions—a process that can disadvantage new firms, which by virtue of their newness are not yet able to establish reputations (Fombrun and Shanley 1990, McGuire et al. 1988). Moreover, organizations tend to engage in exchange with whom they have transacted in the past when market uncertainty is high (Podolny 1994). Therefore, if informational asymmetries are prevalent in that particular market, the new firm may not be able to make itself attractive enough to stakeholders that it can operate.

Lacking their own reputations, young firms may be able to “rent” the reputations of other agents to enhance their own legitimacy and positions in the market. The reputable agent then serves as a middleman between the firm and the stakeholder. In markets where an adverse selection problem exists, the assurance given by a middleman is important to reduce inefficiencies and market failures (Biglaiser 1993). To date, research on young firms has predominately focused on reputable middlemen who provide assurance regarding the firms’ financial attributes. Examples include auditors (e.g., Datar et al. 1991), investment banks (e.g., Tinic 1988, Johnson and Miller 1988, Carter and Manaster 1990) and venture capitalists (e.g., Amit et al. 1998; Amit et al. 1990, 1993).

A central hypothesis of this paper is that reputable outside directors can also provide cues regarding the firms’ attributes (e.g., Pfeffer and Salancik 1978). Indeed, we believe outside directors may be particularly well qualified to efficiently signal information to stakeholders. The advantages of outside directors over inside directors and other possible signalers at communicating information about the potential profitability of firms have two main sources. First, as directors they have access to a great deal of inside information about the firm and the ability to compel its provision. Anyone operating from a position outside the firm would have difficulty securing this kind
of access to important information. Second, they need not be very expensive (relative to the benefit derived) because the amount of time required of even a conscientious outside director is not that great compared, for example, to the time required of a management team.

In addition, we think it is important that, unlike the aforementioned middlemen, reputable outside directors cannot only signal a firm’s financial attributes but also enhance its overall visibility and provide assurance about a wide range of nonfinancial qualities, such as environmental responsibility or ethical conduct. For example, Gilead Sciences, a biotech firm, gained attention by including in their board a then former secretary of defense, Donald Rumsfeld, and a former secretary of state, George Schultz (Stuart et al. 1999). Exxon, an oil company, appointed as an outside director a renowned oceanographer-ecologist, and MP3.com, an online music company, appointed as an outside director a former California Court of Appeals justice. Given the backgrounds of these individuals, we believe the firms may have been trying to signal qualities other than just profitability.

Despite the growing interest in directors’ signaling role (e.g., Certo 2003, Certo et al. 2001, Finkel 1998), to the best of our knowledge no analytical model has been developed to examine this role to date. As a result, we lack understanding when signaling with reputable directors is a feasible strategy and what factors determine its effectiveness. As a first step towards filling this gap, we have developed a signaling model in which new ventures “rent” the reputational capital of outside directors to credibly signal their attributes. Notably, in this model we recognize that, unlike other middlemen (such as venture capitalists, auditors, and investment bankers), outside directors can also be used to signal nonfinancial attributes of the firm. Therefore, in contrast to other adverse selection models, our model does not require that directors’ signal will necessarily separate among firms on the basis of financial performance.

### 3. The Model

Our model is one of adverse selection, in which stakeholders are cautious in aligning themselves with new ventures about which they lack important information. Young firms, on the other hand, need to attract stakeholders (including customers, employees, joint venture partners, etc.) to operate profitably.

We consider a population of new ventures in which there are two types of new ventures, which, for expository convenience, we label “good” and “bad.” A new venture’s type is solely determined here from the stakeholders’ point of view, and will depend upon the attribute the particular stakeholder cares about most. Potential investors, for example, will define a new venture as “good” if its expected profitability is higher than that of other firms, whereas potential employees will define a new venture as “good” if its expected employment conditions are better than those of other firms.

A stakeholder takes some risk in committing to a firm, as doing so requires an irreversible investment. We assume that investments in “bad” new ventures are completely lost, but this is a normalization. The important consideration is that the return is sufficiently low to warrant withdrawal from the commitment. However, both “bad” and “good” firms need to attract stakeholders to make profits. Because stakeholders want to establish working relations only with good firms, both types of new ventures would like to represent themselves as good. For ease of exposition, we assume that each firm needs to attract only one stakeholder.

We assume that the stakeholder does not have enough information to distinguish between good and bad firms. Rather, based upon the information he or she does have, a stakeholder forms a belief about the proportion of good firms in the market. For simplicity, we assume that this belief is correct, but this assumption is not essential.

The model is developed in five steps. In the first step, we describe the basic adverse selection problem, which explains when signaling is needed. In the second step, we introduce directors to the model under the following assumptions: (a) directors have no reputation, (b) directors have complete information regarding new ventures’ types, and (c) good new ventures have higher revenues than bad new ventures do. We relax Assumption (a) in the third step, introducing reputable directors. In the fourth step we relax Assumption (b) and assume that directors have
incomplete information regarding the new venture’s type. In the last step, we relax Assumption (c) and allow the revenue of good new ventures to be lower than that of bad new ventures, permitting “good” and “bad” to refer to something other than profitability.

3.1. The Adverse Selection Problem
Consider potential stakeholders who have to decide whether or not to establish working relations with new ventures. A nonrecoverable investment, I, is needed from the stakeholder to establish such a relationship. The stakeholder’s returns are received from a third party and are dependent on the new venture’s type: good or bad. A relationship with a good new venture provides long-term benefits, A to the stakeholder, whereas the benefits gained from a relationship with a bad new venture are normalized to be zero. The difference in stakeholders’ benefits gained from a relationship with good and bad firms stem from the cost imposed on the stakeholder who attaches himself to a bad firm. An example of such cost incurred by affiliating with a bad firm is damage to that stakeholder’s reputation (Benjamin and Podolny 1999, Stuart et al. 1999).

We assume that good ventures are worthwhile A > I. Stakeholders have (accurate) prior beliefs regarding the proportion of good new ventures (γ) in the population. If stakeholders are capable of screening out some of the bad new ventures, γ is properly interpreted as the fraction of firms that are actually good within the set that survive the initial screening.

If the proportion of good new ventures in the population is high enough, the expected gain to forming a relationship with the firm (γA) will equal or exceed the investment made (I). Stakeholders should thus be willing to gamble on finding a good firm in a random draw from the population of firms if γ ≥ I/A (note that because we expect the benefits, A, to exceed the investment, I, we have 0 < I/A < 1). In this case, all young firms, good or bad, will be able to attract stakeholders. Here, the stakeholders’ lack of information regarding the new venture’s type does not affect the survival rate of new ventures.

When the perceived proportion of good new ventures in the population lies below the critical level (i.e., if γ < I/A), stakeholders will not be willing to invest without more information about firm quality. In the absence of mechanisms to address this situation, the familiar adverse selection problem (see Akerlof 1970) will cause this market to fail. No young firm, good or bad, will be able to attract stakeholders. Clearly, good new ventures have an incentive to try to distinguish themselves from bad new ventures. Note that the critical level of γ will depend on the costs of the stakeholder’s investment (I) and the benefits if the new venture is a good one (A), in fairly obvious ways. The lower the required investment, and the higher the benefit, the higher will be the willingness of the stakeholders to establish relations with new firms.

3.2. Directors as Signaling Vehicles
In the face of a potential market failure, we see a role for the appointment of outside directors (hereafter, simply referred to as directors) to a company’s board of directors as a vehicle by which a good new firm can try to send a credible signal of its quality. However, not all directors have the reputational capital necessary to send credible signals. We begin by considering the use of unknown directors (i.e., those with no reputational capital).

The game begins with new ventures offering directorships to outside agents at a wage, W*. We assume there is a large enough number of potential directors that they have no market power—if they are offered a wage that matches or exceeds their other opportunities (i.e., their reservation wage), they will sign on with the offering firm. The game continues as follows:

**Extensive Form of the Game.**

**Stage I.** The new venture offers the directorship at a wage W* to a potential director. This offer is observed by all players.

**Stage II.** The potential director learns the new venture’s type and decides whether to accept or to reject the offer.1

**Stage III.** The potential stakeholder observes which firms hired directors and decides whether to establish working relations with the new venture.

**Stage IV.** Stakeholders learn the new venture’s type, and players receive their payoffs.

1 There may be a cost associated with informing oneself about the quality of a new venture, but we assume that this cost is part of the director’s reservation wage.
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Figure 1 Timeline of the Game

Firm offers Stakeholder’s directorship decision

join/don’t join

Director’s Stakeholder learns firm’s type decision accept/reject
direction well/bad payoffs received

These stages are summarized in Figure 1.

Signaling models are notorious for the multiplicity of their equilibria (e.g., Rasmusen 2001, Chapter 11). While we restrict ourselves to studying perfect Bayesian equilibria, we will see that there can even be more than one of these in our games depending on the beliefs held by players. Fortunately, we can use a refinement from Cho and Kreps (1987) to select out an equilibrium that has a considerable amount of intuitive appeal (indeed their refinement is referred to as the “intuitive criterion”).

Directors with No Reputation. Assume that potential directors are unknown individuals with no particular reputation. They have a reservation wage ($W$), and are offered a compensation ($W^*$) for holding a director’s position. A good new venture can produce revenue ($R$), whereas bad new ventures can produce revenue ($r$). Both types will produce zero revenue if they do not establish a working relationship with a stakeholder. We assume, for now, that $R > r$ and that a potential director has complete information (i.e., she knows ex ante with certainty the new venture’s type).

As indicated above, when $\gamma \geq 1/A$ all firms are able to attract stakeholders without signaling, and thus, neither type of new venture has an incentive to bear the cost of the signaling. This is summarized in Proposition 1.

**Proposition 1.** In the model in which directors do not have reputations, if $\gamma \geq 1/A$, a pooling equilibrium exists.

Neither good nor bad new ventures will choose to hire outside directors for signaling purposes. All new ventures will be able to attract stakeholders.

From this point on, we address in this model the more interesting case related to conditions under which directors can be helpful as a way to reduce the adverse selection problem. Under certain conditions, good firms may be able to pay such a high premium to directors that bad firms will not be able to match it. While this premium raises the cost of hiring a director, it can also credibly signal the firm’s quality, allowing the good firm to attract stakeholders and operate where this would not otherwise have been possible. This is established in Proposition 2.

**Proposition 2.** In the model with directors, if $\gamma < 1/A$, $R \geq W$, and $R > r$, a separating equilibrium exists. In this equilibrium: (i) good new ventures hire directors and pay them a wage of $W^* = \max\{r, W\}$, (ii) bad new ventures do not hire directors, and (iii) stakeholders sign on only with good new ventures.

The hiring of a director in this case is a means by which a good firm can signal its type by “burning money” in a way analogous to the use of advertising to signal quality in the model of Kihlstrom and Riordan (1984). In this equilibrium, good firms pay so much that bad firms cannot profitably imitate them. The equilibrium is supported by the belief by stakeholders that any director receiving a wage less than $W^*$ must be attached to a bad firm.

It is worth noting two points about Proposition 2. First, in this equilibrium, while investments are all

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2 The proofs of all propositions are available in the Appendix.
ex post profitable for stakeholders, we do not achieve the full-information, first-best outcome, as the signaling is costly. Second, if it were the case that $R \leq W$, the signal would be so costly that no firm would hire a director, and market failure would thus occur.

The separating equilibrium of Proposition 2 is not the only equilibrium possible when $\gamma < 1/\alpha$, $R \geq W$, and $R \geq r$. Depending on the beliefs held by stakeholders, there may also be a pooling equilibrium in which no firm can attract stakeholders. However, as described in Proposition 3, the pooling equilibrium is vulnerable to the refinement developed by Cho and Kreps (1987) and is referred to as the “Intuitive Criterion.”

**Proposition 3.** In the model with directors, if $\gamma < 1/\alpha$, $R \geq W$, and $R \geq r$, a pooling equilibrium also exists in which no firms attract stakeholders. This equilibrium does not satisfy the Cho-Kreps Intuitive Criterion, however.

Essentially, the Intuitive Criterion rules out the pooling equilibrium by arguing that the beliefs that support it are unreasonable. This leaves the separating equilibrium for our continued analysis.

The cost of the signal is determined by $r$ (as long as $r > W$). The higher $r$ is, the more costly the signal will be for the firm that has to pay the director at least $r$ to ensure that bad firms cannot imitate the signal. The following case suggests that hiring reputable directors can reduce the cost of the signal significantly.

**Directors with Reputations.** Next, we assume that directors come with reputations that have been earned from activities outside the model—from a public history of success and truthful speaking, for example. Directors’ reputations act as a type of goodwill—assets that pay dividends to the holders by allowing them to make credible promises that it would not otherwise be possible to make. The promises of “reputable” directors are credible because their reputational capital would be damaged if they did not keep their promises or tell the truth.

A director is chosen based on her reputational capital to provide assurance regarding a certain firm attribute, which a particular group of stakeholders care mostly about. We assume that a director suffers damage to her reputation when it is known to these stakeholders that the director accepted a directorship with what these stakeholders consider a bad firm. Thus, to protect her own reputation, a director adopts these stakeholders’ definition of bad and good firms when deciding whether to accept a directorship. To avoid additional notation, we assume that all of a director’s reputational capital is lost when she attaches herself to a bad firm. Thus, we represent by $P$ both the current level of a director’s reputation and the punishment she suffers by joining a bad firm. The value of $P$ can vary from director to director but it is always observable to all parties. In fact, $P$ must be observable because it is determined by other people’s perceptions.

As in the former case, the director receives compensation $W^*$. In this case, however, we assume that the higher a potential director’s reputation is, the higher will be his or her reservation wage (i.e., $W^*(P)$ is increasing and continuous in $P$). To be willing to sign on with a firm, a director in this scenario must be compensated for not only her time, but also for any cost associated with a possible loss of reputation should the firm turn out to be a bad firm. Therefore, she will demand a wage of $W^*(P) + P$ from a bad firm, and a wage of only $W^*(P)$ from a good firm.

**Proposition 4.** In the model with reputable directors, if $\gamma < 1/\alpha$ and $R \geq W(0)$, a separating equilibrium exists. In this equilibrium: (i) good new ventures hire directors with reputation $P^*$ and pay them a wage of $W^* = r - P^*$ where $P^* = r - W^*(P^*)$, (ii) bad new ventures do not hire directors, and (iii) stakeholders sign on only with good new ventures.

In this equilibrium, the use of directors’ reputations reduces the cost of the signal because the reputations make hiring directors more expensive for bad firms, and bad firms will, therefore, be less able to afford the expense associated with mimicking good firms. Because directors with better reputations are likely to demand higher compensation, the new venture must choose from a range of potential directors the one that provides the needed signal at the lowest cost.

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3 The model allows directors to resign as long as they do so before stakeholders decide whether to establish working relations with the new venture.
The optimal level of reputation ($P^*$) is achieved when $r - P^* = W(P^*)$. Hiring a director with a greater reputation will increase the cost of the signal in one way because it will result in a higher reservation wage. However, the greater is the director’s reputation, the relatively more expensive he is for a bad firm and therefore the less a good firm needs to rely on burning money to distinguish itself from bad firms.

Straightforward comparative static analysis of the equilibrium conditions reveals that

$$\frac{dP^*}{dr} = \frac{1}{1 + W'(P^*)} > 0$$

and

$$\frac{dW^*}{dr} = \frac{W'(P^*)}{1 + W'(P^*)} \geq 0,$$

where $W'(P^*) = \frac{dW(P^*)}{dP^*}$. Therefore, as the profitability of bad firms approaches that of good ones, good firms must employ directors with higher reputations and pay them more.

We make two other points about this proposition. Again, if the signal is too costly to the good firm (i.e., $R < W(0)$), no signal is given and we return to the more complete market failure. Second, the reputation reduces the cost of signaling and achieves a more efficient solution only if at least some directors are not too costly (i.e., $r > W(0)$). If reputable directors are too costly for bad firms, the equilibrium will revert to that of Proposition 2. In the equilibrium of this proposition, investments are ex post profitable for stakeholders and if $r > W(0)$, the cost of signaling is lower than in the “burning money” case.

### 3.3. Potential Directors with Incomplete Information

In the previous step, we assumed that a director could obtain full information about firms (i.e., a director could learn the new venture’s type with certainty ex ante). Now, we relax this assumption, and assume that a director observes the new venture’s type imperfectly. Specifically, we assume that the probability that a director is correct in her assessment is $\alpha$. The probability that a director is mistaken is thus $1 - \alpha$. We also assume that all directors have the same information. All potential directors thus have the same views about any given firm’s type, so a firm rejected by one director cannot keep “shopping” for a director until it finds one who believes it is a good firm.

**Directors Without Reputations.** If directors do not have reputations, their decisions about whether or not to join a firm cannot be affected by concerns about loss of reputation. The consequence is that directors can only be used as a way to burn money, as in Proposition 2. We thus have the following proposition.

**Proposition 5.** When directors have no reputation, the quality of their information about a firm’s type does not affect the separating equilibrium (i.e., it is not affected by $\alpha$). If $\gamma < 1/A$, $R > W$, and $R > r$, a separating equilibrium exists. In this equilibrium: (i) good new ventures hire directors and pay them wage of $W^* = \max(r, W(0))$, (ii) bad new ventures do not hire directors, and (iii) stakeholders sign on only with the good type of firm.

**Directors with Reputations.** Using Bayes’ Rule, we define

$$\beta = \frac{\alpha \gamma}{\alpha \gamma + (1 - \alpha)(1 - \gamma)}$$

to be the probability that a new venture is of the good type, conditional on the director’s information that it is of the good type. When $\alpha = 1$ (i.e., directors have complete information), $\beta = 1$, and when $\alpha = 1/2$ (i.e., directors have no added information to that of stakeholders), $\beta = \gamma$, and therefore $\gamma \leq \beta \leq 1$. Given the potential damage to valuable reputations, reputable directors will demand a premium over their reservation wages to compensate them for the possible loss of reputation, should it turn out that the firm is bad.

If $\alpha$ is high enough, the use of reputable directors by good firms can reduce the cost of signaling. However, good firms can take advantage of this possibility, only when bad firms cannot afford to compensate probability that a director is correct when she believes a firm is good is the same as the probability she is correct when she believes the firm is bad. (In other words, the probabilities of Type I and Type II errors are assumed equal.) This is merely for notational convenience; allowing for different probabilities will require some redefinition of the critical parameters and will be defined below, but will otherwise not affect the analysis.
directors with information that those firms are bad. We use Bayes’ Rule again to define
\[
\delta = \frac{\alpha(1 - \gamma)}{\alpha(1 - \gamma) + (1 - \alpha)\gamma}
\]
as the probability that a new venture is bad given that the director’s information is that it is of the bad type. We then have the following proposition.

**Proposition 6.** In the model with reputable directors, if \(1 > \alpha, \beta \geq I/A \geq \gamma, R > r, \) and \(r > R(1 - \alpha) + \alpha W^*\) (where \(W^*\) is as given below), a stochastic-separating\(^5\) equilibrium exists in which: (i) a proportion of good firms given by \(\alpha\), and a proportion of bad firms given by \((1 - \alpha)\) hire directors and pay them a wage of \(W^* = r - \delta P^*\), where \(P^* = \frac{r - \overline{W}(P^*)}{(1 - \beta + \delta)}\), (ii) \(1 - \alpha\) of the good new ventures and \(\alpha\) of the bad new ventures cannot hire directors at the necessary wage, and (iii) stakeholders sign on only with new ventures that were able to hire directors.

The restriction that \(r > R(1 - \alpha) + \alpha W^*\) is imposed to guarantee that this solution is less expensive for good firms than simply burning money. If this condition does not hold (i.e., \(r < R(1 - \alpha) + \alpha W^*\)) bad firms are so unprofitable that the good firms prefer to distinguish themselves by “burning money.” The equilibrium then is identical to that of Proposition 2.

An important assumption underlying the result in Proposition 6 was that good firms that could not attract directors would be out of luck and unable to operate. We can change this assumption and allow good firms that were not able to recruit a reputable director a second chance to signal their type by burning money (i.e., they could hire a director and offer him or her a wage of \(r\)). If a good firm has such a second chance, the condition \(r > R(1 - \alpha) + \alpha W^*\) is not relevant. Good firms first try to hire reputable directors. If they fail, they can offer directors a wage of \(r\). In this scenario, in equilibrium, all good new ventures are able to hire directors. However, some lucky bad new ventures will successfully hire directors as well.

The contracts offered by good and bad firms when directors have information that they are good must satisfy two conditions: (i) they must be too expensive for bad firms for which directors get correct information to compensate directors for their opportunity costs (including expected damage to their reputations)—this requires \(W^* \geq r - \delta P\), and (ii) they must compensate directors for their opportunity costs (including expected damage to their reputations) \(W^* = \overline{W}(P^*) + (1 - \beta)P^*\).

Here, the minimal cost of signaling is achieved when \(r - \delta P = \overline{W}(P) + (1 - \beta)P\). Because \(I/A < 1\) and the better the information is (i.e., as \(\alpha\) approaches 1) the closer \(\beta\) will be to 1, the condition \(\beta \geq 1/I \geq \gamma\) is likely to be met when the directors have very good information about firm types. It is worth emphasizing that \(\beta\) and \(\delta\) can be close to 1 even for low \(\gamma\) if the information is good enough.

With further analysis of the comparative statics properties of the equilibrium of Proposition 6, we can consider the effect on the equilibrium level of reputation, \(P^*\), of changes in two critical parameters: the accuracy of the directors’ information, \(\alpha\), and the initial proportion of good firms in the population, \(\gamma\).\(^6\) Straightforward calculations reveal that: (i) \(dP^*/dy > 0\), (ii) \(dP^*/d\alpha > 0\) when \(\gamma > 1/2\) and \(dP^*/d\alpha < 0\) when \(\gamma < 1/2\), (iii) \(dP^*/d\gamma \geq 0\), and (iv) if we give the reservation wage the form \(\bar{W}(P^*) = \bar{W} + \bar{W}(P)\), we can demonstrate that \(dP^*/d\bar{W} \leq 0\).

The first condition indicates that as the proportion of good firms in the population of firms is greater, firms that hire directors will bring in directors with greater reputations. The second condition reveals that the effect of improving directors’ information (\(\alpha\)) will depend on the initial distribution of types of firms. This is because the effects on \(\beta\) and \(\delta\) have opposing influences on the equilibrium reputation. Increasing \(\alpha\) will increase \(\beta\) and \(\delta\). The net effect turns on the relative probability of directors seeing a firm as good or bad and this will depend on the initial distribution of types. When good firms are relatively scarce,

\(^5\) The authors thank James Brander for suggesting the label for this type of equilibrium. Note that this equilibrium is different from the “hybrid” equilibrium (see, Gibbons 1992, pp. 202–205) in which one type of agent chooses a pure strategy and the other chooses to randomize. In the stochastic separating equilibrium, derived here, neither type of agent deliberately randomizes—the random outcomes are determined by chance.

\(^6\) The derivations of the comparative statics are available from the authors upon request.
Increasing $\alpha$ has a larger effect on $\beta$ than on $\delta$ resulting in a higher equilibrium level of reputation. And when good firms are relatively common ($\gamma > 1/2$) the reverse is true. The third condition reveals that as the profitability of bad firms approaches that of good ones, good firms will choose directors with higher reputations. The fourth condition suggests that the higher the intercept of the reservation wage of directors, the lower will be the optimal level of reputation.

Figure 2 illustrates the effect of the quality of the directors' information about firm type on the different equilibria offered in this section.

The equilibrium is supported, in part, by stakeholders' beliefs that any firm with a director with $P < P^*$ must be bad. The stochastic-separating equilibrium described here has the appealing property that it allows bad and good firms to coexist in equilibrium. The quality of the directors' information determines the mix of good and bad firms in the population. When directors' information improves, the percentage of good firms in the population rises and $\beta$ shifts closer to 1 (see Figure 2). This continues until the point where directors' information is complete and $\beta$ converges to 1. At this point firms' ventures are 100% of the population, and thus the familiar separating equilibrium is seen to be a special case of the stochastic-separating equilibrium.

When the quality of directors' information falls the percentage of good new ventures in the population decreases and $\beta$ shifts closer to $\gamma$ (see Figure 2). This continues until directors have no added information to that of the stakeholders and $\beta$ converges to $\gamma$. At this point, the only way to signal is by burning money and we reduced to the equilibrium of Proposition 2. We obtain here a result for the poor information case, which is expressed in Proposition 7.

**PROPOSITION 7.** When the quality of directors' information is low (i.e., $\alpha$ is close to $1/2$),7 directors' reputations do not affect the equilibrium. In the model with reputable directors, if $I/A > \beta$, $R \geq W(0)$, and $R > r$, a separating equilibrium exists. In this equilibrium: (i) good new ventures hire directors and pay them wage of $W^* = r$, (ii) bad new ventures do not hire directors, and (iii) stakeholders sign on only with good new ventures.

In this case, the quality of the information directors have about firm type is so poor that we return to the “burning money” result of Proposition 2, in which good firms pay directors a wage of $W^* = r$, and the reputations of those directors are irrelevant to hiring decisions.

### 3.4. What If the Revenue of a Good Type Is Lower than That of a Bad Type (i.e., $R < r$)?

In contrast to other adverse selection models, our model does not require that good new ventures be more profitable than others. Unlike investors, some stakeholders may care more about other attributes of the firm such as its employment conditions or environmental standards than they do about its profitability. We therefore recognize that new ventures which

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7 Given $\gamma$, this critical value of $\alpha$ is easily computed using $I/A = \beta$ and the definition of $\beta$. Note that $\alpha = 1/2$ implies that $\beta = \gamma$, and this proposition become identical to Proposition 1.
stakeholders consider to be good are not necessarily the most profitable. Significantly, different sets of stakeholders may care about different attributes at the same time.

If \( R < r \), burning money is not going to work as a signaling device—bad firms have more money to burn than do good firms. Therefore, if directors have no reputations and \( \gamma < I/A \), neither good nor bad new ventures will be able to attract stakeholders. When directors can be assessed according to reputation, and they have enough information to distinguish good firms from bad, the ability of firms to "rent" directors' reputational capital becomes even more important—it may be the only way for good firms to signal their difference from bad firms. A separating equilibrium exists similar to the equilibrium presented in Proposition 4.

In this case, even when \( R < r \), the separating equilibrium may obtain because the bad new ventures have to compensate directors for the potential loss of reputation. We recognize that signaling to several groups of stakeholders with different interests might demand that the firm hire a number of directors, each with the kind of reputational capital that addresses the interests of a certain group of stakeholders. For example, placing a prominent environmentalist on a board might attract environmentally sensitive customers, but it might not help potential employees wondering about future working conditions. They might be better served by the appointment of another director with a reputation for managing employee-friendly firms.

As with the earlier case, we may observe the stochastic-separating equilibrium seen in Proposition 6 if information is incomplete. In this case, however, the good firms cannot choose to signal their type by burning money and therefore the condition \( r > R(1 - a) + \alpha W^* \) is not required. We thus have the following proposition.

**Proposition 8.** In the model with reputable directors with incomplete information, when \( 1 > \alpha, \beta \geq I/A \geq \gamma, R > W^* \), and \( R < r \), a stochastic-separating equilibrium exists. In this equilibrium: (i) the proportion of good new firms given by \( \alpha \) and the proportion of bad firms given by \( 1 - \alpha \) hire directors at a wage of \( W^* = r - \delta P^* \), where \( P^* = \frac{(r - W(P^*))(1 - \beta + \delta)}{1 - \beta} \), (ii) \( 1 - \alpha \) of the good type and \( \alpha \) of the bad type do not hire directors, and (iii) stakeholders sign on only with new ventures that hire directors.

### 4. Summary and Discussion

This paper has attempted a step toward a better understanding of outside directors' signaling role—a role that is especially important for entrepreneurial firms. The model developed here enables us to formally examine not only whether this strategy is feasible and why the signal is credible, but also factors that determine its effectiveness.

Appointing reputable directors is a compelling signaling vehicle, yet it is not always feasible. When directors have incomplete information regarding the new venture type, the directors must take into account the possibility that they misjudged the new venture type. They may decide to reject the offer position, leading to a market failure. The better directors' information is, the less likely the market failure will occur. We have demonstrated that even directors with incomplete information can serve as middlemen, provided they have some information to be added to that of the stakeholders.

Because directors are likely to command greater compensation, the better their reputation, the new venture must choose from a range of potential directors with different reputational capital—the one who provides the needed signal at lowest cost. Four critical parameters in our analysis affect the optimal level of directors' reputation: (i) the proportion of good firms in the new venture population, (ii) the quality of directors' information, (iii) the revenue of the bad firms, and (iv) directors' reservation wage. The higher the proportion of good firms in the population and the higher the revenue of the bad firms, the higher will be the optimal level of directors' reputation. In contrast, higher directors' reservation wage leads to a lower optimal level of directors' reputation. The effect of directors' information depends on the percentage of good new ventures in the population. When the majority of new ventures are good, the higher the quality of directors' information, the higher the optimal level of their reputation. Whereas, if the majority of the firms in the population is of the bad type, the better the quality of directors' information, the lower the optimal level of directors' reputation.
Our model suggests that directors can also signal attributes of the firm other than profitability. This finding hints at the possibility of using directors as a complementary signal to those of other agents, such as investment banks and auditors. Because the latter are most likely to signal financial attributes of the firm, outside directors with different reputations can provide assurance regarding various nonfinancial attributes (e.g., environmental responsibility, ethical conduct). A young firm, for example, may choose to signal its favorable financial prospects with a reputable investment bank, and at the same time, provide assurance about its environmental responsibility by appointing an outside director with the relevant reputation. Admittedly, inside directors, and not only outside directors, could credibly be used to signal a nonfinancial firm attribute. However, firms are likely to prefer using an outside director rather than a full-time executive of the firm because hiring the outside director is less expensive.

The results derived from our model can also inform the broader signaling literature. We believe that there is something novel in the stochastic-separating equilibrium described here. This type of equilibrium is obtained when the middleman’s (the director’s) information regarding the firm quality is incomplete. On one hand, the stochastic-separating equilibrium can be seen as a pooling equilibrium because both types of new ventures choose the same action: to offer the directors the wage of \( W^* \). On the other hand, it resembles a separating equilibrium in the way that the directors serve to distinguish, to some extent, the good new ventures from the bad. This equilibrium concept may well be applicable in a broader set of models that incorporate signaling through middlemen. It has an important realistic feature in that it permits the coexistence of both high and low types of firms in equilibrium without pure pooling. The quality of the middlemen’s information determines the mix of these types in the population. The familiar separating equilibrium can be seen in this context as a special case of the stochastic-separating equilibrium, obtained when the middlemen’s information is complete.

This paper offers interesting avenues for future research. First, we have derived a set of testable propositions for future empirical investigation. An important challenge here will be disentangling the signaling function of outside directors from their other function (e.g., monitoring management on behalf of shareholders). A direct way to do this would be through careful survey research in which entrepreneurs who had recently put together a board of directors were asked what motivated the selection of each director. They could be asked directly if a particular appointment was an attempt to signal something, and if so, what. Hedonic regression analysis might also be useful to determine if similarly qualified directors were paid more to serve on the boards of new firms, which would be evidence that they were being paid a reputation-risk premium. Second, our model deals with directors with a one-dimensional reputational capital. An interesting extension could be to examine how having multidimensional reputations, which could possibly provide conflicting signals to stakeholders, will affect stakeholders and directors’ decisions whether or not to join the firm. Third, future research may also help us understand when an outside director’s signal is used as a substitute to other reputable agents’ signal (e.g., reputable auditors, venture capitalists, top management team members) and when it is used as a complementary one. The recent trend toward compensating outside directors with incentive pay may affect their credibility as middlemen because incentive pay may offset the potential loss of reputation suffered by the directors who attach themselves to the bad firms.

The results of our analysis have important normative implications for practitioners. The main implication is that by hiring reputable directors, high-quality young firms may enhance their survival prospects. Signaling efforts should be directed towards stakeholders who need to make a large relationship-specific investment in establishing working relations with the young firms, and whose benefits from these relationships are not expected to be extremely high. However, because we recognize that boards have various roles other than signaling, we do not imply that an entire board should be structured for signaling purposes. Rather, one (or perhaps a few) individual(s) with the relevant reputational capital can be hired as signals. Furthermore, because directors who are
more reputable are likely to demand higher compensation, firms should not hire the most reputable director available but the one whose reputation is sufficient to provide the needed signal. Finally, firms should bear in mind that it is not sufficient to hire reputable directors to provide a credible signal. It is also necessary to provide the director with high-quality information regarding the firm. By providing directors the high-quality information, firms can not only increase the probability they will attract stakeholders, but also reduce their signaling cost.

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Appendix

PROOF OF PROPOSITION 1. The proof proceeds by demonstrating that in the proposed equilibrium, every agent (and agents of each type) is optimizing subject to the strategies of the other agents.

The expected profit of the stakeholder \( E(I_1) = \gamma A - I \) when \( \gamma \) is nonnegative and stakeholders are willing to sign on with any new venture selected at random. The good new ventures' payoff: \( I_1 = R - W^* \) is maximized when \( W^* = 0 \) (i.e., when they do not hire directors). The bad type's payoff: \( I_1 = r - W^* \) is also maximized when \( W^* = 0 \).

The stakeholder could not be doing any better, because staying out of the game offers no benefits, whereas choosing randomly a new venture offers a positive expected benefit. The good new ventures have no incentive to buy a costly signal because the stakeholder will establish working relations in any case. The bad new ventures have no incentive to distinguish themselves from the good ones. The potential directors who are not offered any wage would not participate.

PROOF OF PROPOSITION 2. The equilibrium is supported by beliefs on the part of stakeholders that if they observe the signal (the hiring of the director at a wage \( W^* > r \)), the new venture is good, and if they do not observe the signal, the new venture is bad. In this equilibrium the payoffs are as follows:

\[ \begin{align*}
\text{The stakeholder's payoff: } & \quad I_s = A - I > 0. \\
\text{The good new venture's payoffs: } & \quad I_v = R - W^*. \\
\text{The bad new venture's payoffs: } & \quad I_b = 0. \\
\text{The director's payoff: } & \quad I_d = W^* \text{ (or, net of opportunity cost, } W^* - W). 
\end{align*} \]

The stakeholder could not be doing any better because the signal costs nothing and provides full information. The good new venture is making a positive profit, which would fall to zero if it chooses not to signal because potential stakeholders would infer that it was a bad type and would not sign on. The director's wage of \( W^* \) cannot be lower than \( r \), because it must prevent the bad type from imitating the signal. This wage must also be higher than the director's reservation wage. Therefore, \( W^* = \max(r, W) \). The bad firm is shut out of this market and makes zero profits, as the cost of the signal is greater than the revenue it would derive by attracting the stakeholder (i.e., \( W^* \geq r \)). The directors will sign on because \( W^* \geq W \). Therefore, every agent is optimizing, and only the good new ventures attract stakeholders.

PROOF OF PROPOSITION 3. The pooling equilibrium is supported—as in many signaling models—by stakeholder beliefs that the signals are meaningless. When stakeholders feel this way, there is reason for anyone to pay for a signal. The direct result, under the parameter configuration of the proposition, is that no stakeholders sign up with firms and no firms can operate. Using the arguments of Cho and Kreps (1987), however, we argue that these beliefs are not reasonable and so this equilibrium will not be relevant. To understand this refinement, consider the stakeholder's response if he expected the pooling equilibrium but then observed a new venture deviate by offering a director a wage \( W^* > r \). What should the stakeholder infer about a firm taking this unexpected action? The Intuitive Criterion asks us to consider which type could possibly benefit from such a deviation. This suggests that in this case the stakeholder should infer that the firm must be a good one because a bad firm could not profit from such a deviation; only a good firm could afford a wage that high. If stakeholders will make such an inference, good firms will deviate this way and the pooling equilibrium is broken. According to the Intuitive Criterion then, the pooling equilibrium in this setting is not reasonable. (For a textbook treatment of the Intuitive Criterion, see Fudenberg and Tirole 1992, pp. 446–456.)

PROOF OF PROPOSITION 4. As in the former case, the equilibrium is supported by beliefs on the part of stakeholders that if they observe the signal, the new venture is of the good type, otherwise it is of the bad type. In this equilibrium the payoffs are as follows:

\[ \begin{align*}
\text{The stakeholder's payoff: } & \quad I_s = A - I > 0. \\
\text{The good new venture's payoffs: } & \quad I_v = R - W^*. \\
\text{The bad new venture's payoffs: } & \quad I_b = 0. \\
\text{The director's payoff: } & \quad I_d = W^* \text{ (or net of reservation wage } = W^* - W(P)). 
\end{align*} \]

The stakeholder could not be doing any better because the signal costs nothing and provides full information. The good new venture is making a positive profit, which would fall to zero if it chose not to signal because stakeholders would infer it was a bad type and would not sign on. The director's wage of \( W^* \) should be
higher than \( r - P \) to ensure that the bad type cannot imitate the signal. It must also be higher than \( W(P) \) or directors will not participate. Because the good type maximizes its payoff when \( W^* \) is minimized, it would choose \( P \) such that \( r - P = W(P) \). Any change in \( P \) will cause an increase in \( W^* \). The bad new ventures are shut out of this market and make zero profits. They cannot hire a director because the director’s wage \( W^* \) plus the required compensation for the director’s potential loss of reputation \( P \) is greater than the revenue the bad type would derive by attracting the stakeholder. Directors will sign on with the good firms because \( W^* > W(P) \). They would not sign on with the bad new venture if the offered wage does not compensate for the potential loss of reputation (i.e., \( W^* < W(P) \)). Therefore, every agent is optimizing.

Proof of Proposition 5. When directors have no reputation, they have no reputation to lose (i.e., \( P = 0 \)), and are thus indifferent between holding directorship in good new ventures and in bad new ventures. The proof, therefore, is similar to the proof of Proposition 2.

Proof of Proposition 6. The equilibrium is supported by beliefs on the part of stakeholders that directors would accept a directorship only in firms that directors perceive as good new ventures. In this equilibrium, the expected payoffs are as follows:

The stakeholder’s expected payoff: \( E(\Pi) = (A - 1)\beta - l(1 - \beta) \).

The good new venture’s payoffs if it hires a director: \( \Pi = R - W^* \).

The bad new venture’s payoffs: \( \Pi_b = r - W^* \). (Both types’ payoff is zero if a director is not hired.)

The director’s expected payoff: \( E(\Pi_d) = W^* - (1 - \beta)P = W(P) \).

The stakeholder could not be doing any better because the signal costs nothing and the expected payoff is positive. The good type’s expected profit is positive, but would fall to zero if it did not hire a director. The good type will choose to use directors with reputation as long as the expected payoff is higher than using directors with no reputation and paying them a wage of \( r \) (i.e., \( \alpha(R - W^*) + (1 - \alpha)0 > R - r \)). Rearranging this condition, we get \( r > R(1 - \alpha) + \alpha W^* \). Note that when \( W^* = r \), a separating equilibrium exists, in which all good new ventures operate. In the stochastic-separating equilibrium, a proportion of the good firms given by \( (1 - \alpha) \) does not survive and, therefore, gain zero payoff.

A director’s wage of \( W^* \) must not be lower than \( r - 3\delta P \) to ensure that directors will not choose to sign on with new ventures that the directors perceive as bad. \( W^* \) must also not be lower than \( W(P) + (1 - \beta)P \), otherwise the directors would refuse to participate. Because the good new ventures maximize their payoff when \( W^* \) is minimized, they would choose \( P \) such that \( r - 3\delta P = W(P) + (1 - \beta)P \). Therefore, any change in \( P \) will cause an increase in \( W^* \). The bad firm will offer the same wage if \( r > W^* \), otherwise its payoff will be zero. Bad new ventures will successfully hire directors only when the directors erroneously perceive them as good. Therefore, every agent is optimizing.

Proof of Proposition 7. The stakeholders would participate only if their expected payoff \( E(\Pi) \) is positive. Because \( E(\Pi) = (A - 1)\beta - l(1 - \beta) \geq 0 \), stakeholders will participate only if \( \beta \geq l/A \).

When the director’s compensation is \( r \), the director’s reputation does not affect the equilibrium. Therefore, this equilibrium will be identical to the equilibrium in Proposition 2 (see Proof of Proposition 2).

Proof of Proposition 8. The proof here is similar to the Proof of Proposition 7. In this proposition, however, because \( r > R \), it is not possible to signal by burning money. Therefore, the condition \( r > R(1 - \alpha) + \alpha W^* \) is not relevant.

Proof of Comparative Statics. Recall that

\[
P^* = \frac{r - W(P^*)}{1 - 3 + 3},
\]

where

\[
\hat{\alpha} = \frac{\hat{\alpha}(1 - \hat{\alpha})}{\hat{\alpha}(1 - \hat{\alpha}) + (1 - \hat{\alpha})}, \quad \hat{\alpha} = \frac{\hat{\alpha} + (1 - \hat{\alpha})}{\hat{\alpha}(1 - \hat{\alpha}) + (1 - \hat{\alpha})}.
\]

Let \( 1 - \hat{\alpha} = D \) so

\[
P^* = \frac{r - W(P^*)}{D}, \quad \frac{DP^*}{D} = r - W^*(P^*).
\]

Total differentiation reveals

\[
D \frac{dP^*}{d\alpha} = -W^* \frac{dP}{d\alpha} \quad \text{and} \quad D \frac{dP^*}{d\alpha} = -W^* \frac{dP}{d\alpha},
\]

so

\[
\frac{dP}{d\alpha} = \frac{-P}{D + W^*} \frac{dD}{d\alpha}, \quad \frac{dD}{d\alpha} = \frac{-P}{D + W^*} \frac{dD}{d\alpha},
\]

\[
\frac{dD}{d\alpha} = -\alpha(1 - \alpha) \frac{dD}{d\alpha} \quad \text{and} \quad \frac{dD}{d\alpha} = -\alpha(1 - \alpha) \frac{dD}{d\alpha},
\]

\[
= \frac{\alpha(1 - \alpha)}{[\alpha + (1 - \alpha)(1 - \gamma)]^2}, \quad \text{so} \quad \frac{dD}{d\alpha} = -\alpha(1 - \alpha) \frac{dD}{d\alpha}.
\]

Thus, we see that

\[
\frac{dD}{d\alpha} = \frac{-P}{D + W^*} \frac{dD}{d\alpha} > 0.
\]

Then, for

\[
\frac{dD}{d\alpha} = -\frac{\alpha}{\alpha + (1 - \alpha)(1 - \gamma)} \frac{dD}{d\alpha} + \frac{\alpha}{\alpha + (1 - \alpha)(1 - \gamma)} \frac{d\delta}{d\alpha} + \frac{\gamma(1 - \gamma)}{(\alpha + (1 - \alpha)(1 - \gamma))^2}
\]

we see that

\[
\frac{dD}{d\alpha} > 0 \quad \text{if} \quad \alpha + (1 - \alpha)(1 - \gamma) > \alpha(1 - \gamma) + (1 - \alpha)\gamma,
\]

or if

\[
\alpha(1 - \alpha) < (1 - \alpha)(1 - \alpha),
\]

because \( \alpha > 1/2 \), by assumption, we know that \( \alpha < (1 - \alpha) \), so \( 1 - 2\gamma < 0 \Rightarrow \gamma > 1/2. \)
Thus,
\[
\frac{dD}{da} > 0 \text{ if } \gamma > \frac{1}{2}, \quad \frac{dD}{da} < 0 \text{ if } \gamma < \frac{1}{2},
\]
and
\[
\frac{dP}{da} = \left( -P \frac{dD}{D+W} \right) \frac{dD}{da}.
\]

Thus,
\[
\frac{dP}{da} < 0 \text{ if } \gamma > \frac{1}{2}, \quad \frac{dP}{da} > 0 \text{ if } \gamma < \frac{1}{2}.
\]

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